1 Introduction

In servo control, two fundamental problems are the point-to-point control problem and the tracking (path following) control problem. The point-to-point control problem is concerned with moving the control object from one point to another. In this problem, the controller is required to provide a small final positioning error and superior regulation. A regulation function is required to keep the object at a desired position in the presence of disturbances. In tracking control, the control object must be moved along the desired trajectory. Tracking control is extremely important in many mechanical systems: e.g., automated arc welding using a robot arm and high speed machining of complex shaped workpieces. The design of controllers for mechanical systems must consider (1) stringent performance specifications in terms of tracking error and speed, (2) nonlinear characteristics such as Coulomb friction, stiction and actuator saturation, (3) time varying characteristics, and (4) the use of microprocessors in implementation. The control algorithms presented in this paper directly address the first and fourth points. The second and third points become important in applying the algorithms to actual systems.

This paper describes my work on the design of digital tracking controllers over the past twenty years. In view of the fourth point mentioned above, I will describe discrete time control algorithms only. My involvement in tracking control goes back to my Ph.D. dissertation, "On Finite Preview Problems and Its Application to Man Machine Systems" (1973). When the desired output is known in advance, i.e., "previewable," the controller does not have to be constrained to work only on the error signal. Prior information can be utilized in a variety of ways to recover the inherent limitation in tracking, i.e., dynamic lag of the control object. One of my doctoral dissertation committee members, T. B. Sheridan, had three models of preview control (1966), but a good model from a viewpoint of optimal control was missing. In the early 1970s, the optimal tracking problem in the linear quadratic optimal control context was well understood (e.g., Athans and Falb, 1966; Anderson and Moore, 1971). However, its solution required the knowledge of future information related to the desired output signal over the entire problem duration. In the finite preview problem (Tomizuka, 1973; Tomizuka and Whitney, 1975; Tomizuka et al., 1984), the basic assumptions were (1) future information is available only in a local sense, i.e., over a time interval, \([k < i < k + N_p] \quad (k + N_p < N)\), where \(k\) is the present time, \(N_p\) is the preview length and \(N\) is the problem duration, and (2) the optimality must be judged based on a performance index defined over the problem duration. More recently, I introduced the idea of zero phase error tracking control (ZPETC) as a generalization of feedforward control based on the idea of inverse systems (1987). The ZPETC can be applied to systems whose inverses are unstable. Other researchers have utilized future values of a desired output for attaining an improved tracking performance, e.g., independent tracking and regulation approach (Landau and Lozano, 1981) and an adaptive LQ controller (Samson, 1982).

In some applications, such as computer disk file systems, the tracking error, that is, the difference between the desired output and actual output, may be the only signal available for the controller (Chew and Tomizuka, 1989) and the desired output itself is not known in advance. However, it may be known that the desired output possesses a certain known property, e.g., it is periodic with a known period. Such properties can be utilized to design controllers which may adsorb the tracking error in an asymptotic sense. The repetitive controller (Tomizuka, Tsao and Chew, 1989) is such a controller. As far as I known, repetitive control was originated by Japanese researchers (Inoue et al., 1981; Omata et al., 1985). My graduate students and I have devoted a significant amount of effort to better understanding and establishing methodologies for repetitive control over the past decade because periodic desired outputs and disturbances are quite common in mechanical systems.

In the following section, the design of tracking controllers for known (i.e., previewable) desired outputs will be discussed. The design of controllers which are solely based on the tracking error will be discussed in Section 3. Conclusions and further remarks including future research directions will be given in Section 4.
2 Tracking Control for Known Desired Output

When the desired output is known in advance, its future values should be utilized to enhance the tracking performance. The overall control system normally takes a two-degree-of-freedom structure in Fig. 1. The feedback controller must be designed to provide an adequate level of regulation as well as robustness. The feedforward controller is used as a prefilter for the desired output signal to compensate for the dynamic lag of the closed-loop systems and to achieve a high tracking performance. The most intuitive prefilter for the purpose is the mathematical inverse of the closed-loop system for perfect tracking control (PTC), explained below.

Let the discrete time transfer function for the closed-loop system be expressed as

\[ G_{\text{closed}}(z^{-1}) = \frac{z^{-d}B_c(z^{-1})}{A_c(z^{-1})} \]

where \( z^{-d} \) denotes a one sampling delay, \( z^{-d} \) represents a d-step delay normally caused by the delay in the plant and computation,

\[ B_c(z^{-1}) = b_{c0} + b_{c1}z^{-1} + \cdots + b_{cm}z^{-m}, \quad b_{c0} \neq 0 \]
\[ A_c(z^{-1}) = 1 - a_{c1}z^{-1} - \cdots - a_{cn}z^{-n} \]

Our goal is to let the output \( y(k) \) follow the desired output \( y_d(k) \). To achieve this objective, the reference input for the closed-loop, \( r(k) \), is determined by processing \( y_d(k) \) by

\[ G_{ff}(z^{-1}) = G_{\text{closed}}(z^{-1})^{-1} = \frac{z^{d}A_c(z^{-1})}{B_c(z^{-1})} \]

so that the overall transfer function from \( y_d(k) \) to \( y(k) \) is unity. Equation (2) implies that the closed-loop poles and zeros should be canceled by the zeros and poles of the feedforward controller. Notice that this feedforward controller is unrealizable form, and it must be implemented as

\[ r(k) = A_c(z^{-1}) B_c(z^{-1}) y_d(k + d) \]

Equation (3) clearly indicates the necessity of previewing the desired output: i.e., the desired output must be known in advance. This requirement is not unrealistic in many servo problems such as control of machining centers.

\[ G_{ff}(z^{-1}) \text{ in Eq. (2) is unstable and is unusable if any closed-loop zero appears on or outside the unit circle in the z-plane; in digital control, it is well known that the unstable discrete time zero(s) appear when fast sampling is applied to a continuous time plant with a relative degree greater than or equal to two (Astrom and Wittenmark, 1984). For example, a pure double integrator plant \( G_p(z) = 1/z^2 \) generates a zero at \( -1 \). The ZPETC described later in this section was developed to deal with unstable or cancellable zeros of the closed-loop system. The prefilter can be designed based on the linear optimal control theory instead of pole/zero cancellation. While the PTC and ZPETC approaches attempt perfect or near perfect tracking of the desired output, the optimal preview approach minimizes combinations of the tracking error and control effort. Chronologically, the tracking controller design based on optimal control precedes the development of the design based on the mathematical inverse, though the latter appears to be more intuitive and straightforward.

2.1 Tracking Controller Design Based on Optimal Control

The optimal tracking control problem was formulated as a natural extension of the linear quadratic (LQ) optimal control and can be found in a number of standard text books. The continuous time optimal tracking problem can be found in Athans and Falb (1966) and the discrete time version can be found in Anderson and Moore (1971). Since the optimal tracking problem is the basis of other optimal tracking controllers, it is summarized below.

2.1.1 Optimal Tracking Problem. In the discrete time optimal tracking problem, the controlled plant is described by

\[ x(k + 1) = A x(k) + B u(k) \]
\[ y(k) = C x(k) \]

where \( x(k) \), \( u(k) \) and \( y(k) \) are the \( n \)-dimensional state vector, \( m \)-dimensional input vector and \( r \)-dimensional output vector, respectively, and \( A \), \( B \), and \( C \) have appropriate dimensions. Given a desired output sequence \( \{y_d(k)\}_{0 \leq k \leq N} \), the optimal control \( u_{opt}(k) \) must minimize the quadratic performance index

\[ J = e^T(N)S e(N) + \sum_{i=0}^{N-1} \{e^T(i)Q e(i) + u^T(i)Ru(i)\} \]

where \( e(k) = y_d(k) - y(k) \), \( N \) is the specified problem duration, \( S \) and \( Q \) are positive semidefinite, and \( R \) is positive definite.

The solution to this optimal control problem is given by

\[ \begin{align*}
    u_{opt}(k) &= -[B^T H(k+1)B + R]^{-1}B^T H(k+1)A x(k) \\
    & \quad \quad \quad \quad \quad \quad \quad + g(k + 1)
\end{align*} \]

where \( H(k) \) is the solution of the Riccatti equation,

\[ H(k) = A^T H(k+1) - H(k+1)B^T H(k+1)B + R]^{-1}B^T H(k+1)A + C^T Q C \]

and \( g(k) \) is given by

\[ g(k) = [A - B B^T H(k+1)B + R]^{-1} B^T H(k+1) A] g(k + 1) - C^T Q y_d(k) \]

Equations (8) and (10) can be solved by off-line backward recursion. Notice that the performance index (6) trades off the tracking error against the input magnitude. It is straightforward to incorporate other quantities such as incremental changes in other state variables and/or control inputs in the performance index.

2.1.2 Optimal Preview Problem. Equation (7) implies that \( u_{opt}(k) \) depends on \( \{y_d(i)\}_{k-i \leq N} \), which is not practical especially when \( N \) is large. It is often the case that the desired output is known in advance only in a local sense: i.e., \( \{y_d(i)\}_{k-i \leq k+P_d, \ k+N_p < N} \) is known, where \( P_d \) is the preview (or look ahead) steps. For the situation where the knowledge on the desired output is local, Hayase and Ichikawa (1969) suggested formulating the optimal tracking problem over the local time interval and applying its solution in a successive manner. However, their solution was only suboptimal when viewed over the entire problem duration, \( N \). The major motivation for the finite preview control theory (Tomizuka, 1973) was to find the tracking control law, which depends only on local future values of the desired output, and yet is optimal in a certain sense over the entire problem duration. The optimal preview control approaches are reviewed below.
The stationary solution is practically important and it minimizes,

\[ J = E \left[ e^T(N)S e(N) + \sum_{i=0}^{N_p-1} [e^T(i) Q e(i) + u^T(i) R u(i)] \right] \tag{14} \]

where the expectation is taken over the random quantity, \( w_d(k) \).

The problem is now well formulated. A more general formulation and its solution, which include the effects of measurement delay and noise, are in Tomizuka and Whitney (1975). The stationary solution is practically important and it minimizes,

\[ J' = \lim_{N \to \infty} J/N \tag{15} \]

where \( J \) is given by Eq. (14). It is written as

\[ u^{opt}(k) = -G_d x(k) + \sum_{i=1}^{N_p} G_{yd}(l) y_d(k+l) + G_{gd} x_d(k+N_p) \tag{16} \]

The first term in Eq. (16) is the feedback control term and the feedback gain is given by

\[ G_d = [B^T H_d B + R]^{-1} B^T H_d A \tag{17} \]

where \( H_d \) is the steady-state solution of the Riccati Eq. (8). The second and third terms in Eq. (16) are the preview control terms and provide feedforward control. \( G_{yd}(l) \)'s satisfy the relation (Tomizuka and Whitney, 1975),

\[ G_{yd}(l) = [B^T H_d B + R]^{-1} B^T H_d y_d(l+1) \tag{18} \]

\[ H_d(I - l) = [A - B G_d]^{-1} H_d(l) \tag{19} \]

where \( A = A - B G_d \) is the closed-loop system matrix for the controlled plant under the feedback control. Since the closed-loop system is asymptotically stable under normally satisfied conditions, Eq. (19) implies that the future desired output \( y_d(k+l) \) has diminishing importance as \( l \) is increased. The third term in Eq. (16) may require the use of an estimator since \( x_d \) is not necessarily accessible. However, the magnitude of \( G_{gd} \) is small for a large \( N_p \). Note that the output of the preview controller is a moving average of a future desired output sequence and that the number of preview steps required to achieve almost all of the possible benefits of preview depends on the eigenvalues of \( A \), or \( Q \) and \( R \) matrices in the performance index (14).

The properties of the preview control terms as explained above imply that any reasonable assumption for \( y_d(i) \) is acceptable when \( N_p \) is sufficiently large, and has motivated deterministic preview control approaches (Tomizuka and Rosenthal, 1979; Tomizuka et al., 1980). In particular, it has been found to be practical to assume that the desired output does not change beyond the preview time interval: i.e., \( y_d(i + 1) = y_d(i) \) for \( i > k + N_p \), where \( k \) is the present time.

### 2.2 Tracking Controller Design Based on Pole/Zero/Phase Cancellation

In the tracking controller design based on optimal control, the tracking error was traded off against the input magnitude. However, in mechanical systems control, the error is of major concern especially when the desired output is precisely known in advance. In such cases, the idea of inverse systems discussed at the beginning of this section is more appealing than the optimal preview controller. As pointed out already, such controllers must deal with unstable or uncancellable zeros of the controlled plant. In order to deal with such zeros, we factorize \( B_c(z^{-1}) \) into two parts and write

\[ B_c(z^{-1}) = B_+ (z^{-1}) B_-(z^{-1}) \tag{20} \]

where \( B^{-1}_c(z) \) contains uncancellable zeros, which include unstable zeros, and \( B^*_c(z^{-1}) \) contains cancelable zeros.

The ZPET feedforward controller (Tomizuka, 1987) is

\[ G_{ZPET}(z^{-1}) = \frac{\frac{d}{dz} A_c(z^{-1}_e) B^{-1}_c(z)}{B^*_c(z^{-1}) B_+(z^{-1})} \tag{21} \]

where \( B^*_c(z) \) is obtained by replacing every \( z^{-1} \) in \( B_c(z^{-1}) \) by \( z \). \( G_{ZPET}(z^{-1}) \) is unrealizable, and this feedforward control law is implemented in the following form:

\[ r(k) = \frac{A_c(z^{-1}_e) B^{*,*}_c(z^{-1})}{B^*_c(z^{-1}) B_+(z^{-1})} y_d(k + d + s) \tag{22} \]

where \( B^{*,*}_c(z^{-1}) = z^{-s} B_c(z \) and \( s \) is the order of \( B^{-1}_c(z) \), that is, the number of uncancellable zeros.

From Eqs. (1) and (22), for zero initial conditions, the plant output is

\[ y(k) = \left[ B^{-1}_c(z^{-1}) B^{-1}_c(z)/B_+(z^{-1}) \right] y_d(k) \tag{23} \]

The following relation can be easily verified by direct substitution of \( z = e^{s \omega T} \), \( 0 \leq \omega \leq \pi/T \) where \( T \) is the sampling period.

\[ Im [e^{s \omega T} B^*_c(e^{s \omega T})/B_+(e^{s \omega T})] = 0, \quad 0 \leq \omega \leq \pi/T \tag{24} \]

Equation (24) implies that, when \( y(k) \) is sinusoidal, there is no phase shift between \( y_d(k) \) and \( y(k) \). The ZPETC cancels the poles and cancelable zeros of the closed-loop system and compensates for phase shifts induced by uncancellable zeros. Because of this property, \( G_{ZPET}(z^{-1}) \) is called the zero phase error tracking controller (ZPETC). The ZPETC requires \( d + s \)-step preview of the desired output while the number of preview steps to attain almost all of the benefit of preview in the optimal preview control depended on \( Q \) and \( R \) matrices in Eq. (14).

Notice that \( B^*_c(1)^2 \) is used as a scaling factor in the ZPET in Eq. (22). The reason for this scaling factor is that in many mechanical systems, uncancellable zeros appear in the left half side of \( z \)-plane and the frequency responses of building blocks of zero phase error transfer functions, \( (z^{-1} - \xi) (z^{-1} - \gamma) \), satisfy the properties summarized in the following theorem.

**Theorem 1.** For any zero in the left-half side of \( z \)-plane, \( \zeta \),
A vehicle lateral guidance in highway automation (Peng and Pak and Tuner, 1986). The continuous time version of the optimal preview controller has been successfully applied to robotic systems (Tomizuka and Janczak, 1985; Tomizuka et al., 1980; Tomizuka et al., 1984) and the use of a frequency shaped performance index (Peng and Tomizuka, 1991).

The effectiveness of ZPETC for mechanical systems has been demonstrated on mechanical systems such as a vertical machining center (Suzuki and Tomizuka, 1991; Tung and Tomizuka, 1992), direct drive robot manipulators (Horowitz et al., 1987) and a high speed positioning table (Endo et al., 1992). In these applications, a superior tracking performance was obtained by first compensating nonlinear characteristics and various uncertainties of the mechanical systems and then applying the ZPETC. For the machining center problem, nonlinear friction forces had to be percompensated so that the dynamics of the closed-loop system could be approximated by a linear transfer function. Furthermore, Tung and Tomizuka (1992) have shown that a ZPET controller can be effectively designed based on a low-order closed-loop transfer function model, which captures the low frequency dynamics. In an extreme case that a first-order model is sufficient for covering the frequency range of interest, the feedforward controller design is simplified because the location of zeros is no longer an issue. ZPETC acts as an excellent feedforward controller for direct drive robot arms, the nonlinear dynamics of which are compensated by adaptive control (Horowitz et al., 1987).

3 Tracking Control for a Class of Unmeasurable Desired Output

Tracking controllers in the previous section require that the desired output is known in advance and is given to the controller independent from the actual plant output. When this is not the case, the controlling input must be based on the tracking error.

Let the controlled plant be described by

\[ A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k) + d(k) \]

where \( d(k) \) is the disturbance,

\[ B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}, \quad b_0 \neq 0 \]

\[ A(z^{-1}) = 1 - a_1 z^{-1} - \cdots - a_N z^{-N} \]

In terms of the tracking error, \( e(k) = y(k) - y_d(k) \), the system is represented as

\[ A(z^{-1})e(k) = z^{-d}B(z^{-1})u(k) + w(k) \]

where

\[ w(k) = d(k) - A(z^{-1})y_d(k) \]

In this section, we consider a class of \( w(k) \), i.e., a class of desired outputs and disturbances, characterized by

\[ W(z^{-1}) = A_d(z^{-1}) \]

The characteristic roots of \( A_d(z^{-1}) \) are normally on the unit circle. For example, Eq. (35) implies that \( y_d(k) \) is a step signal for \( A_d(z^{-1}) = 1 - z^{-1} \), a ramp signal for \( A_d(z^{-1}) = (1 - z^{-1})^2 \), or a periodic signal for \( A_d(z^{-1}) = 1 - z^{-N} \) where \( N \) denotes the period.

Without the loss of generality, it is assumed that Eq. (32) is asymptotically stable. This assumption implies that an appropriate stabilizing controller has been applied when the plant is unstable.

It is also assumed that \( B(z^{-1}) \) and \( A_d(z^{-1}) \) are coprime. Under these assumptions, a variety of approaches are possible for the design of controllers which assure

\[ \lim_{k \to \infty} e(k) = 0 \]
One approach is to utilize the internal model principle (Francis and Wonham, 1975) and to construct the feedback control system as shown in Fig. 3. \( A_d(z^{-1}) \) in the controller is the internal model.

Notice that for the closed-loop system in Fig. 3, \( e(k) \) is

\[
e(k) = \frac{S(z^{-1})A_d(z^{-1})}{S(z^{-1})A_d(z^{-1})A(z^{-1}) + z^{-d}R(z^{-1})B(z^{-1})}w(k)
\]

(37)

where the initial condition has been ignored.

Recall that the characteristic roots of \( A_d(z^{-1}) \) are normally on the unit circle. Therefore, in view of the closed-loop transfer function in Eq. (37), the closed-loop system cannot be made asymptotically stable unless \( A_d(z^{-1}) \) and \( B\left(z^{-1}\right) \) are coprime; notice that common characteristic roots of \( A_d(z^{-1}) \) and \( B\left(z^{-1}\right) \) become closed-loop poles. Under the coprimeness assumption, the closed-loop system can be stabilized; for example, an arbitrary pole placement is possible based on the general property of the Diophantine equation (Astrom and Wittenmark, 1984),

\[
S(z^{-1})A_d(z^{-1})A(z^{-1}) + z^{-d}R(z^{-1})B(z^{-1}) = D(z^{-1})
\]

(38)

Furthermore, noting that the cancellation of \( A_d(z^{-1}) \) takes place in Eq. (37) for \( w(k) \) characterized by Eq. (35), the asymptotic regulation or tracking (36) is achieved. These results have been further refined to the discrete time repetitive controller in Tomizuka et al. (1989).

### 3.1 Discrete Time Repetitive Controller

The repetitive controller is for periodic desired outputs and disturbances with a known period: namely, \( A_d(z^{-1}) = 1 - z^{-N} \) where the period, \( N \), is known. Notice that the internal model for this case is a periodic signal generator. In the development of the repetitive controller, the following points required special attention:

1. **Amount of Real Time Computation.** In repetitive control, \( N \) can be large. While \( A_d(z^{-1}) = 1 - z^{-N} \) in the denominator of the controller can be efficiently computed, the amount of real time computation becomes too excessive if the order of \( R(z^{-1}) \) is high, which is the case if the closed-loop poles are arbitrarily assigned and the Diophantine Eq. (38) is solved for \( R(z^{-1}) \) and \( S(z^{-1}) \). In Tomizuka et al. (1989), it has been suggested to set the controller in the following form.

\[
G_d(z^{-1}) = \frac{R(z^{-1})}{(1 - z^{-N})S(z^{-1})} = k_r z^{-N+d+s} A(z^{-1}) B^{-s}(z^{-1})
\]

\[
(1 - z^{-N}) B^{-s}(z^{-1}) B^s(z^{-1})
\]

\[
\beta = \max_{\omega} B(z^{-1} e^{-j\omega T})
\]

(39)

where \( B^s(z^{-1}) \) and \( B^{-s}(z^{-1}) \) are similarly defined as for the ZPETC. The amount of real time computation to implement (39) depends on the plant complexity, but not on \( N \). The control gain, \( k_r \), must be between 0 and 2 for asymptotic stability of the repetitive control system.

2. **Stability Robustness.** The internal model, \( 1 - z^{-N} \), possesses \( N \) characteristic roots all on the unit circle, which is the stability boundary for the discrete time system. This makes the stability of the overall system highly sensitive to unmodeled dynamics of the controlled plant. The robustness problem has been resolved by modifying the internal model so that the characteristic roots, especially high frequency characteristic roots, are shifted to the inside of the unit circle. Strictly speaking, the modified internal model is no longer a periodic signal generator. In particular, Tsao and Tomizuka (1988) have shown that the following internal model offers flexibility in trading off the tracking performance against robustness.

\[
A_d(z^{-1}) = 1 - q(z, z^{-1}) z^{-N}
\]

(40)

where

\[
q(z, z^{-1}) = \alpha_0 + \alpha_1 z^{-1} + \cdots + \alpha_m z^{-m}
\]

(41)

Notice that \( q(z, z^{-1}) \) is a low pass filter possessing the zero phase shift characteristics. Since the implementation diagram for the repetitive controller with the modified internal model has not been given elsewhere, it is shown in Fig. 4.

### 3.2 Applications of Discrete Time Repetitive Control

The discrete time repetitive control algorithm has been successfully applied to computer disk file systems (Chew and Tomizuka, 1990), noncircular machining (Tsao and Tomizuka, 1988) and robot manipulators (Tsai et al., 1988). Notice that the repetitive controller has a learning ability, so that the system performance is improved each time the controller experiences a repetitive situation. This property has been utilized by Tung et al., (1991) to learn an appropriate control signal for overcoming low velocity friction, for which it is hard to make a precise model.

### 4 Conclusions and Further Remarks

In this paper, the design of digital tracking controllers has been presented for the following two cases: (1) the desired output is known in advance, and (2) the desired output is not directly measured but is known to possess a certain property. For the first case, the design can be based on either the linear optimal control theory (optimal preview controller) or pole/zero/phase cancellation (ZPETC). For the second case, the internal model based controller can achieve asymptotic regulation and tracking of the desired output. An interesting special case of the internal model based controller is the discrete time repetitive controller. Summary of these control algorithms are given below.

- The optimal preview controller minimizes the performance index which includes the tracking error term and the input term. The weights of these terms determine the number of
The optimal preview controller, ZPETC and discrete time repetitive controller have been successfully tested on a variety of mechanical systems.

Some further remarks are

- In tracking control discussed in this paper, the desired output has been given as a time sequence. Such a sequence may be obtained by decomposing a desired spatial trajectory to servo axes. In this case, the desired output sequence for each axis depends on the tracking speed along the trajectory. The tracking problem is more complicated when the tracking speed is adjustable. Huang and Tomizuka (1990) applied the fuzzy set theory to deal with such situations. This type of tracking is called self-paced tracking (Sheridan, 1966). It is an interesting area for further research.

- The optimal preview controller and generalized predictive control (GPC) (Clark et al., 1987) have some common aspects. The optimal preview controller emphasizes the improvement of tracking performance by utilizing the future values of the desired output, and the GPC is motivated by attaining robustness by incorporating the predicted future system behavior. In fact, it is possible to formulate the two problems in a unified mathematical framework (Egami et al., 1992; Konno et al., 1992).

- ZPETC is sensitive to modeling errors. The frequency spectrum of the desired output is normally band limited, and it is important to develop a mathematical model of the closed-loop plant in the relevant frequency range. It should be noted that the tracking performance under ZPETC depends on the mathematical model as well as on identification algorithms for model development (Tung and Tomizuka, 1992).

- The discrete time repetitive control algorithm based on the internal model principle is one of the possible approaches for dealing with periodic signals. For other approaches, see Kempf et al. (1992).

As the title of my dissertation (Tomizuka, 1993) suggests, I was a member of the Man Machine Systems Laboratory of MIT and worked on preview control from a human machine systems point of view. After joining the University of California at Berkeley, I have found that “tracking” is one of the most important features for mechanical systems such as a robot manipulator and a machining center. In tracking arbitrary shaped desired outputs, the system remains continuously transient and never reaches the steady-state, which is a unique aspect in mechanical systems control and provides a challenge to mechanical control engineers.

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