Chapter II – 3  Pumps for Pipelines

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II.3.1. Energy Relations for a Pumping System

Extended Bernoulli equation written between the reservoir free surfaces;

\[
\frac{P_{et}}{\rho g} + \frac{V_{et}^2}{2g} + Z_{et} + H - \sum_{j=1}^{N} h_j = \frac{P_{bt}}{\rho g} + \frac{V_{bt}^2}{2g} + Z_{bt}
\]

Total head Loss in the System

\(j\) : Element with head Loss (Pipe, elbow, valve, etc.)

For Large Reservoirs:

\[
\frac{V_{et}^2}{2g} \approx \frac{V_{bt}^2}{2g} \approx 0
\]

If reservoirs are open to atmosphere;

\[
\frac{P_{et}}{\rho g} = \frac{P_{bt}}{\rho g}
\]

Typical Pumping System

\[\begin{align*}
P_s &: Pump suction end Pressure \\
P_d &: Pump discharge end Press. \\
P_{sr} &: Suction Reservoir Press. \\
P_{dr} &: Discharge Reservoir Press. \\
H_{gt} &: Total geometric head \\
H_{gd} &: Discharge geometric head \\
H_{gs} &: Suction geometric head \\
H_{gp} &: Pump geometric head
\end{align*}\]
II.3.1. Energy Relations for a Pumping System

Extended Bernoulli equation written between the reservoir free surfaces;

Total Geometrical Head $H_{gt}$

$$H_{gt} = Z_{bt} - Z_{et}$$

Thus the Pump Head; $H$

$$H = H_{gt} + \sum_{j=1}^{N} h_j$$

$$H = H_{gt} + h_t$$

Pump Head “$H$” required in a system between two large reservoirs open to atmosphere is equal to the sum of total geometric head “$H_{gt}$” and total head loss in the system “$h_t$”

Typical Pumping System

- $P_s$: Pump suction end Pressure
- $P_d$: Pump discharge end Press.
- $P_{sr}$: Suction Reservoir Press.
- $P_{dr}$: Discharge Reservoir Press.
- $H_{gt}$: Total geometric head
- $H_{d}$: Discharge geometric head
- $H_{gs}$: Suction geometric head
- $H_{gp}$: Pump geometric head
II.3.1. Energy Relations for a Pumping System

Total head loss in a system can be expressed as the sum of total head losses in the suction(s) and discharge(d) sides.

\[
\sum_{j=1}^{N} h_j = \sum_{j=1}^{N} (h_j)_s + \sum_{j=1}^{N} (h_j)_d
\]

Extended Bernoulli equation written between the suction and discharge sides of the pump gives the total pump head, \( H \).

\[
\frac{p_{1s}}{\rho g} + \frac{V_{1s}^2}{2g} + Z_{1s} + H = \frac{p_{2d}}{\rho g} + \frac{V_{2d}^2}{2g} + Z_{2d}
\]

\[
H = \left( \frac{p_{2d}}{\rho g} + \frac{V_{2d}^2}{2g} + Z_{2d} \right) - \left( \frac{p_{1s}}{\rho g} + \frac{V_{1s}^2}{2g} + Z_{1s} \right)
\]
**II.3.1. Energy Relations for a Pumping System**

For small pumps or when the suction and discharge side of a pump are at the same geometrical head and of equal pipe diameter; the pump head can be measured and the differential static pressure between them.

\[ H = \frac{p_2 - p_1}{\rho g} \]

The Power transformed to the fluid is the product of Specific Weight, \((\gamma \equiv \rho g)\); volumetric flowrate, \(Q\) and total pump head \(H\), and referred to as Hydraulic Power \(P_h\).

\[ P_h = \rho g Q H \]

The effective power to operate the pump, \(P_{eff}\) is referred to as; **brake horse power (bHP)** or is **Shaft Power**. The shaft power can be expressed in terms of the shaft toque and speed as;

\[ P_{sh} = \omega T \quad \omega : \text{shaft rotational speed} \]

\[ T : \text{torque} \]
II.3.1. Energy Relations for a Pumping System

As the shaft power is transformed to hydraulic power, some portion of it is lost due to various reasons, resulting in a smaller hydraulic power. The losses expressed as the ratio of input to output powers is expressed as follows defining the pump efficiency $\eta$.

$$\eta = \frac{P_H}{P_m} = \frac{\rho g QH}{\omega T}$$

The losses in a pump are:

- Losses in the mechanical transmission system losses (shaft, bearing, sealing and windage). These are expressed in terms of the mechanical efficiency ($\eta_m$).
- Fluid losses inside the pump (impeller, diffuser, volute, casing) due to friction turbulence and separation. These are expressed in terms of the hydraulic efficiency ($\eta_h$).
- Leakage losses between the impeller and the casing. These are expressed in terms of the volumetric efficiency ($\eta_v$).

The pump designer aims to design a pump which has the highest efficiency in a wide range of flowrates.
II.3.1. Energy Relations for a Pumping System

The pump efficiency (overall) is the product of these three efficiencies.

\[ \eta = \eta_m \cdot \eta_v \cdot \eta_h \]
II.3.3 Dimensional Analysis and Similitude in Turbomachinery

• Turbomachinery performance is a function of many physical parameters. Some of these parameters are geometric, some are dynamic and others are fluid property parameters.

• Full performance can hardly be analytically expressed through these parameters, so an empirical approach is necessary to obtain the characteristics of a turbomachine.

• A “Dimensional Analysis” is very useful in reducing the number of parameters especially reducing the number of experiments and in compacting the results.

• Here the non-dimensional parameters (π terms) effective in performance are derived for a general turbomachine to express the performance non-dimensionally; but more important to establish similarity of operation between various cases and benefit in applying others empirical results to our cases of interest.
II.3.3 Dimensional Analysis and Similitude in (Pumps) Turbomachinery

• The physical parameters of interest are given below

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>Volumetric flowrate</td>
<td>m³/s</td>
</tr>
<tr>
<td>gH</td>
<td>Energy per unit weight</td>
<td>m²/s²</td>
</tr>
<tr>
<td>D</td>
<td>Size of the turbomachine, Impeller diameter</td>
<td>m</td>
</tr>
<tr>
<td>Ω</td>
<td>Rotational speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>P</td>
<td>Power of the turbomachine</td>
<td>W</td>
</tr>
<tr>
<td>ρ</td>
<td>Fluid density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>μ</td>
<td>Fluid viscosity</td>
<td>Pa·s</td>
</tr>
</tbody>
</table>

Power of the turbomachine can be expressed as:

\[ P = (Q, gH, D, \omega, \rho, \mu) \]
II.3.3 Dimensional Analysis and Similitude in Pumps

A dimensional analysis method such as Buckingham pi ($\pi$) theorem can be used to obtain the non-dimensional parameters as:

\[ \pi_Q = \frac{Q}{\omega D^3} \]
\[ \pi_H = \frac{gH}{\omega^2 D^2} \]
\[ \pi_P = \frac{P}{\rho \omega^3 D^5} \]
\[ \pi_{Re} = \frac{\mu}{\rho \omega D^2} \]

Here the 7 physical parameters effective in turbomachinery performance are expressed in terms of 4 $\pi$ terms as non-dimensional performance parameters.

\[ \frac{P}{\rho \omega^3 D^5} = \left( \frac{Q}{\omega D^3}, \frac{gH}{\omega^2 D^2}, \frac{\mu}{\rho \omega D^2} \right) \]

Power parameter  Flow parameter  Load parameter  Reynolds Number
II.3.3 Dimensional Analysis and Similitude in Pumps

In geometrically similar machines full similitude can be obtained if all \( \pi \) terms are the same. This can be hardly satisfied, so the equality of Reynolds number is relaxed. The error made due to this is later corrected by empirical correlations. Thus for similitude;

\[
\frac{P}{\rho \omega^3 D^5} = \left( \frac{Q}{\omega D^3}, \frac{gH}{\omega^2 D^2} \right)
\]

Thus, for the similitude of geometrically similar turbo-machines it is enough to have the flow and load parameters to be the same in both machines.
II.3.3 Dimensional Analysis and Similitude in Pumps

The performance parameter efficiency as previously defined may be obtained by modifying the $\pi$ terms.

\[
\pi_{\eta_P} = \frac{\pi_Q \pi_H}{\pi_P} = \frac{Q gH}{\omega D^3 \omega^2 D^2} = \frac{\rho gQH}{\rho \omega^3 D^5} = \eta_{pump}
\]

Pump Efficiency

\[
\pi_{\eta_t} = \frac{\pi_P}{\pi_Q \pi_H} = \frac{P}{\rho \omega^3 D^5} = \frac{Q gH}{\omega D^3 \omega^2 D^2} = \frac{P}{\rho gQH} = \eta_{turbine}
\]

Turbine Efficiency

For a pump (or turbine) operating in similitude disregarding Reynolds Number effects the efficiencies are the same.
II.3.4. Specific Speed and Typification

When the performance of geometrically similar pumps (of different sizes) are expressed in terms of similarity parameters, they collapse to a single curve. Any deviation from this is due to:

- **Measurement errors**
- **Scale factor effect**
- **Reynolds number effect**

Experimental Results

Eliminating $D$ between $\pi_Q$ and $\pi_H$ a ND-parameter independent of size factor can be obtained.

\[
N_s = \frac{\pi_Q^{\frac{1}{2}}}{\pi_H^{\frac{3}{4}}} = \left( \frac{Q}{\omega D^3} \right)^{\frac{1}{2}} = \left( \frac{gH}{\omega^2 D^2} \right)^{\frac{3}{4}} = \frac{\omega Q^2}{(gH)^{\frac{3}{4}}}
\]
II.3.4. Specific Speed and Typification

For a given type (family) of turbomachinery the typifiying “Specific Speed $N_s$” is defined, based on the $H$ and $Q$ values at the best efficiency operating point (design point $*$).

Thus the Specific Speed typifying the pump $N_s$ is defined based on the $H$, $Q$ and $N$ values at the best efficiency (design) point designated by ( )$*$ is expressed as;

$$N_s = \frac{2\pi \left( N^*[rpm] / 60 \right) \left( Q^*[m^3/s] \right)^{1/2}}{\left( g[m/s^2] \right)^{3/4}}$$
II.3.4. Specific Speed and Typification

Specific speed is the type parameter corresponding to the required flowrate and Head to designate the shape of the pump giving the highest efficiency.

Pump typification according to $N_S$

Turbine Typification according to $N_{SP}$
II.3.4. Specific Speed and Typification

The Specific Speed is used in a dimensional form in practice.

For Pumps

\[
N_s = \frac{\omega Q^2}{(H)^{\frac{3}{4}}}
\]

\(\Omega\): rotational speed (rad/s)

For Turbines

\[
N_{sp} = \frac{\pi}{\rho^{\frac{5}{4}}} \left( \frac{P}{\rho \omega^3 D^5} \right)^{\frac{1}{2}} = \frac{\omega P^2}{\rho^{\frac{5}{4}} (gH)^{\frac{5}{4}}}
\]

\(N_{sp}\) is the Power Specific Speed
Pump performance characteristics are obtained through extensive tests. Characteristics expressed in terms of non-dimensional parameters help the user to compare the types of pumps without including the effect of size factor. In the figure a comparison of 3 different pumps having specific speeds;

$N_s = 0.4 \rightarrow$ Centrifugal,
$N_s = 3.0 \rightarrow$ Mixed Flow,
$N_s = 5.8 \rightarrow$ Axial Flow, are given for comparison.
II.3.5. Performance Characteristics

Figure: Stable and Unstable Pump Characteristics

Figure: Surging Pumps with Unstable Characteristics
II.3.5. Performance Characteristics

**Figure**: Three Dimensional Pump $\phi$ (H-Q-$\eta$)

**Figure**: Two Dimensional Pump $\phi$ (H-Q-$\eta$ & NPSH-Req.)
II.3.5. Performance Characteristics

Figure: Volumetric (Positive Displacement) Pump Characteristic

Figure: Turbine Characteristics
II.3.5. Performance Characteristics

II.3.5.a Operating Point

Operating Point (OP) is defined as the intersecting point of Pump Characteristic and System Characteristic.

Figure: Pump Characteristic, System Characteristic & Operating Point
Change of Operating Point due to Changes in System Characteristics in Rotodynamic Pumps

1- System 𝜉 Change due to Geometrical Head ($H_{gt}$) Variations
1. System Change due to Geometrical Head (Hgt) Variations
II.3.5.a Operating Point - Rotodynamic Pumps

2- System change due to Valve Throttling \((CQ^2\text{ Variations})\)
2- System change due to Valve Throttling ($CQ^2$ Variations)
II.3.5.a  Operating Point - Positive Displacement Pumps

Change of Operating Point due to Changes in System Characteristics in Positive Displacement Pumps

1. System change due to Geometrical Head ($H_{gt}$) Variations

![Diagram showing change in pump characteristic due to geometrical head variations]
II.3.5.a Operating Point - Positive Displacement Pumps

2. System $\phi$ Change due to Valve Throttling ($CQ^2$ Variations)
In practice the non-dimensional parameters are simplified to form simpler but dimensional performance parameters.

\[
\pi_Q' = \frac{Q}{N D^3}
\]

Flow parameter

\[
\pi_H' = \frac{H}{N^2 D^2}
\]

Load parameter

N : rpm

These can be used as similarity parameters.
II.3.3.1 Simplified Non-Dimensional Groups

II.3.3.1.a Speed Change in Pumps

As the rotational speed of the pump $N$ changes, the Pump Characteristic changes. As a consequence the Operating Point changes. The Pump $\phi$ at this new $N$ can be found through similitude. If the change in $N$ is too large, a Reynolds Number correction may be necessary. For this case the simplified similarity parameters for pumps are:

$$
\pi_Q' = \frac{Q}{N} \quad \pi_H' = \frac{H}{N^2} \quad \pi_P' = \frac{P}{N^3}
$$

These dimensional $\pi'$ terms have to be the same for similar operating points at two different $N$ values.

$$
\frac{Q_1}{N_1} = \frac{Q_2}{N_2} \quad \frac{H_1}{N_1^2} = \frac{H_2}{N_2^2} \quad \frac{P_1}{N_1^3} = \frac{P_2}{N_2^3}
$$
If the pump characteristic at $N_1$ is known, the characteristic at $N_2$ could be found using the similarity parameters.

If $N$ is eliminated between these 2 similarity relations, a relation between $H$ and $Q$ can be found. This is the “Similarity Curve for geometrically equivalent (same) pumps operating at different speeds.

$$H = \frac{\pi'_H}{\left(\frac{\pi'_Q}{\pi_Q}\right)^2}Q^2 = CQ^2 \rightarrow C = \frac{H}{Q^2}$$

This relation gives similar operating points at different $N$ values.

The efficiencies of similar operating points can be taken to be the same.
II.3.3.1  Simplified Non-Dimensional Groups
II.3.3.1.a Speed Change in Pumps

Figure: Similar Operating Points in Pumps with Speed Changes. Similar Operating Points
Flowrate Adjustment by Speed Control in a Pumping System

Diagram showing a pump performance curve, with points OP1, OP2, OPo, and OPo connected to (H1, Q1), (H2, Q2), (Ho, Qo), and (Ho, Qo) respectively. The curves represent different speeds N1 and N2.
II.3.3.1 Simplified Non-Dimensional Groups

II.3.3.1.a Diameter Change in Pumps

For Geometrically similar pumps the Pump impeller diameter, D can be changed, keeping the pump speed, N constant. If the geometric similarity is not disturbed by this diameter change, the similarity parameters (“affinity laws”) may be used to obtain the modified characteristics. Thus:

\[ \pi_Q'' = \frac{Q}{D^3} \quad \pi_H'' = \frac{H}{D^2} \quad \pi_P'' = \frac{P}{D^5} \]

The \( \pi'' \) terms given above are dimensional. For the similar operating points of two different diameters they have to be the same.

\[ \frac{Q_1}{D^3_1} = \frac{Q_2}{D^3_2} \quad \frac{H_1}{D^2_1} = \frac{H_2}{D^2_2} \quad \frac{P_1}{D^5_1} = \frac{P_2}{D^5_2} \]

If the characteristics of a pump of diameter \( D_1 \) at speed N is known, the characteristics of another geometrically similar pump of diameter \( D_2 \) operating at the same speed N can be found through the above relations.
II.3.3.1 Simplified Non-Dimensional Groups

II.3.3.1.a Diameter Change in Pumps

From these relations if “D” is eliminated, a relation between H and Q may be obtained.

\[ \pi_H'' = \frac{H}{D^2} \quad \pi_Q'' = \frac{Q}{D^3} \]

\[ H = \left( \frac{\pi_H''}{\pi_Q''} \right)^{\frac{2}{3}} Q^{\frac{2}{3}} = CQ^{\frac{2}{3}} \]

\[ C = \frac{H}{Q^2} \]

This relation gives the locus of similar operating points for geometrically similar pumps of different size (D), but operating at the same rotational speed (N).
II.3.3.1 Simplified Non-Dimensional Groups
II.3.3.1.a Diameter Change in Pumps

Figure: Similar OP in Similar Pumps with Diameter Change
Paralel Pipeline Systems (Single Pump – 2 Reservoirs)
Paralel Pipeline Systems (Single Pump – 2 Reservoirs)

\[ H_2 = H_{gt2} + K_2 Q^2 \]
\[ H_1 = H_{gt1} + K_1 Q^2 \]
• Overloading Pumps
• Pump Combinations
  Series and Parallel Pumps
• System Combinations
• Flowrate Control
• Cavitation