COMSOL\textsuperscript{1} Tutorial 1

**Problem Definition**

$$-\frac{d^2u}{dx^2} - u = -x^2, \quad 0 < x < 5$$

$$u(0) = 0, \quad u(5) = 10$$

Exact solution of the problem is known as

$$u_{\text{exact}} = 2\cos(x) + x^2 - \frac{\sin(x)[2\cos(5) + 13]}{\sin(5)} - 2$$

**Why This Problem?**

To demonstrate the unique ability of COMSOL that allows you set up your own DEs to define new problems.

\textsuperscript{1} COMSOL 4.3 is used to prepare this tutorial. It should be available at the PC labs operated by the Computer Center.
1. Start COMSOL.

In the “Model” tab select “1D” and click the “Next” arrow.

Select “Coefficient Form PDE (c)” and click the “Next” arrow.
Select “Stationary” and click the “Finish” button.
2. In the “Model Builder” (MB) tab right click “Geometry 1” and select “Interval”. An item called “Interval 1” will be created in the MB.

In the “Interval” tab, set “Left endpoint” and “Right endpoint” to 0 and 5, respectively.

Click “Build Selected” in the “Interval” tab.

You’ll see that the 1D domain will spear as a line in the “Graphics” tab.
3. In the MB tab select “Coefficient Form PDE 1”. You’ll see the general equation that COMSOL uses. To put the general equation into the form that we want to solve, change
\[ c = 1, \]
\[ a = -1, \]
\[ f = -x^2 * 1 \text{ [m}^4\text{]} \]
**Note:** \([1 \text{ [m}^4\text{]}\) part is necessary to make the unit of \(f\) 1/m^2
and set all other coefficients to zero.
4. By default both boundary conditions are set to zero flux (NBC type). To change them right click “Coefficient Form PDE 1” in the MB tab and select “Dirichlet Boundary Condition”. A new item named “Dirichlet Boundary Condition 1” will appear in the MB tab.

On the “Graphics” tab use the mouse to select the left boundary point.

In the “Dirichlet BC” tab press “Add to selection” button.

Do not change the default value of \( r = 0 \), which is the value specified at the boundary.
To change the second BC, right click “Coefficient Form PDE 1” in the MB tab and select “Dirichlet Boundary Condition”. A new item named “Dirichlet Boundary Condition 2” will appear in the MB tab.

On the “Graphics” tab use the mouse to select the right boundary point.

In the “Dirichlet BC” tab press “Add to selection” button.

Set $r = 15$. 
5. On the MB tab select “Mesh 1”.
In the “Mesh” tab, do not change the default settings for “Sequence type” and “Element size”.
Click “Build All”.

As will be seen in the “Messages” tab, a mesh of 15 elements will be generated. By changing the parameters of the “Mesh” tab we can control the number and distribution of the elements.
6. Select “Study 1” in the MB tab and press the “Compute” button in the “Study” tab.
Solution will finish in a couple of seconds and the following variation of the unknown will be generated. Use the “Image Snapshot” button to copy the plot to the clipboard to paste it into a document or save it as an image file.

When the solution is done “1D Plot Group 1” item will appear in the MB tab under “Results”. Many properties of the above plot, such as its title, axes names, line color, etc. can be changed by selecting it.
7. It is possible to post-process the solution data in many different ways. For example one common thing we do is to evaluate the secondary variable (or the gradient that appears in the secondary variable) at boundary points where EBC is specified.

In this problem EBC is specified at the left boundary. To calculate the slope of the unknown there, in the MB tab right click “Derived Values” under “Results” and select “Point Evaluation”.

In the Graphics tab select both the left and right boundary points using the mouse and the Ctrl key. Press “Add to Selection” button of the “Point Evaluation” tab.

Change “Expression” to “ux” which means the x derivative of u. To see other available derived values, click the “Replace Expression” button of the “Expression” part.

Click the “Evaluate” button at the top of the “Point Evaluation” tab.

The calculated gradient of u at the boundaries will appear in the Results tab.
8. To generate a new plot right click the “1D Plot Group 1” under “Results” of the MB tab. A new item called “Line Graph 2” will appear.

In the Graphics tab select the 1D domain and click the “Add to Selection” button of the “Line Graph” tab.

In the “Line Graph” tab, change “Expression” to “ux” to plot the gradient of u.

Press the Plot button.
Both \( u \) and \( du/dx \) will be seen in the plot as follows

To see the plot of only \( du/dx \), in the MB tab right click “Line Graph 1” and select “Disable”. Plot of \( u \) will disappear. You can enable it again any time you want.

Or for \( du/dx \) you could have created a new “1D Plot Group” by right clicking “Results” in the MB tab and only show \( du/dx \) on it.
9. To export the solution to a text file right click “Solution 1” under “Results -> Data Sets” in the MB tab and select “Add Data to Export”.

A new “Data 1” item will appear under Export. Select it.

Add “u” and “ux” to the Expression list.

Select a file name and press the Export button.
The following text file will be created with $u$ and $\frac{du}{dx}$ values at 16 mesh points.

| % Model: | 1D Mathematics.mph |
| % Version: | COMSOL 4.3.0.233 |
| % Date: | Mar 10 2015, 06:21 |
| % Dimension: | 1 |
| % Nodes: | 16 |
| % Expressions: | 2 |
| % Description: | Dependent variable $u$, Gradient of $u$, x component |
| $x$ | $u$ (1) | $u_x$ (1/m) |
| 0 | 0 | 14.27520268327608 |
| 0.3333333333333333 | 4.630211434650534 | 13.49807723901236 |
| 0.6666666666666666 | 8.764994833797937 | 11.305964753953269 |
| 1 | 11.985870425070749 | 8.016022636887183 |
| 1.3333333333333335 | 13.9994285656260318 | 4.063811190991501 |
| 1.66666666666666672 | 14.669628987292304 | -0.04220385942013789 |
| 2.0000000000000001 | 14.032777744747994 | -3.7766254358683966 |
| 2.3333333333333334 | 12.293529433250153 | -6.654963018071798 |
| 2.6666666666666668 | 9.802358476174408 | -8.286967565305783 |
| 3.0000000000000018 | 7.016377322256469 | -8.419591485646673 |
| 3.33333333333333353 | 4.451949417006243 | -6.9648455430567004 |
| 3.6666666666666669 | 2.622049060788285 | -1.0094848148221764 |
| 4.0000000000000003 | 1.9855682498962945 | 0.1945152114397338 |
| 4.3333333333333337 | 2.89391452084759 | 5.237825885213952 |
| 4.66666666666666705 | 5.552977324218402 | 10.69636992377012 |
| 5 | 10 | 15.96893454496697 |
Exercises:

1. Add a new Line Graph to “1D Plot Group 1” under results and use it to plot the known exact solution. For the Expression directly enter the known $u_{\text{exact}}$ as a function of $x$. You’ll see that the FEM solution and the exact solution are on top of each other.

2. Change the BCs as follows and solve the problem again

$$u(0) = 10 , \quad \frac{du}{dx}\bigg|_{x=5} = 15$$

You need to change the value of the first Dirichlet BC and change the type of the second BC to Flux/Source.

Check the slope of the FEM solution at $x = 5$ and see whether it is exactly equal to 15 or not. After performing a new solution the derived values shown in the Results tab will not be updated automatically. You need to evaluate them. To avoid confusion clear the table before a new evaluation to get rid of the old values.

3. Change the BCs as follows and solve the problem again

$$u(0) = 10 , \quad \left(u + \frac{du}{dx}\right)\bigg|_{x=5} = 15$$

Again in COMSOL the second BC is of Flux/Source type.