1) A single degree of freedom physical model of a quarter-car active suspension system is given in Figure 1. The masses of the car chassis and the wheel assembly are combined into the equivalent mass \( m \), whose position is expressed by \( x(t) \). The passive elements of the suspension are represented by an equivalent spring \( k \), and an equivalent damper \( b \). A valve-controlled electro-hydraulic actuator is utilized as the active control element in the suspension system, where the manipulated input is the spool position \( x_v \). The road disturbance is denoted by \( y(t) \).

![Physical model of a quarter-car active suspension system](image-url)
The aim of this study is to obtain a mathematical model of the system, where the controlled output is the velocity of the mass to which the driver’s seat is assumed to be rigidly connected, \( \dot{x}(t) = v(t) \), and to design an open loop speed controller to improve the driver’s comfort in addition to an existing closed loop controller. Two ideal accelerometers measure the accelerations of the chassis, \( \ddot{x}(t) \), and the road disturbance, \( \ddot{y}(t) \).

a) As the initial step of the control system design and analysis, the mathematical model of the system is obtained by writing the elemental and structural equations in Laplace domain in the following table. Complete missing equations.

<table>
<thead>
<tr>
<th>Elemental Equations:</th>
<th>Structural Equations:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hydraulic</strong></td>
<td><strong>Mechanical</strong></td>
</tr>
<tr>
<td>( Q_L = K_vX_v - K_pP_L )</td>
<td>( F_m = ... )</td>
</tr>
<tr>
<td>( Q_c = ... )</td>
<td>( F_k = ... )</td>
</tr>
<tr>
<td>( F_b = ... )</td>
<td>( F_m = ... )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Hydraulic Actuator</strong></th>
<th><strong>Q_p = ...</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_p = ... )</td>
<td>(6)</td>
</tr>
</tbody>
</table>

b) Complete the block diagram representation of the open loop system given in Figure 2.

![Block diagram of the open loop system](image)
c) By using only the block diagram reduction techniques, determine the plant and disturbance transfer functions, $G_u(s)$ and $G_d(s)$, respectively, as shown in Figure 3.

\[ \begin{align*}
&G_u(s) \\
&s^2Y(s) \\
&G_d(s) \\
&X_Y(s) + sX(s) = V(s)
\end{align*} \]

**Figure 3 Simplified open loop block diagram**

d) Design a typical open loop control system by determining $G_r(s)$ and $G'_d(s)$. Next, draw a complete block diagram of the combined open loop and closed loop control system by assuming that a proportional controller $K_p$ is already designed and available in the closed loop system. Do not forget to include the transfer functions of the accelerometers in your block diagram.

e) Knowing that the velocity reference input to this control system is zero, $V_r(s) = 0$, for improved driver comfort, discuss the effectiveness of the open loop control system.

2) Consider the car given in the figure. While travelling on the road, car’s velocity changes due to wind, inclination on the road, and the traction force supplied by the engine. You need to design a combined controller for the car to travel at a constant speed (a simple cruise controller).

Assume that
- There is no slip between tires and the road, so absolute velocity $v_c$ of the car should be measured from ideal wheel speed sensor,
- The relative velocity $v_{w/c}$ of the wind with respect to car is measured with an ideal flow meter placed on the car,
- The inclination angle is measured with an ideal sensor (inclinometer) as percent grade of the ramp $\beta$,
- The force due to inclination is proportional to percent grade of the road as $f_r = K_R \beta$. 


The traction force is proportional to throttle opening as \( f_t = K_T \theta_T \).

The air drag force is viscous and it is proportional to the relative velocity of the wind with respect to car.

Note that in real cruise control system, flow meter or inclination sensors are not used. Instead, they usually work based on a velocity feedback of the car, and an embedded controller.

<table>
<thead>
<tr>
<th>( m ): Car mass</th>
<th>( \beta ): percent grade of ramp</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_c ): Absolute velocity of car</td>
<td>( b_w ): viscous drag coefficient of wind</td>
</tr>
<tr>
<td>( v_w ): Absolute velocity of wind</td>
<td>( v_{w/c} ): wind velocity relative to car</td>
</tr>
</tbody>
</table>

The block diagram representation of the car is shown below. The throttle opening is the input and the velocity of the car is the output of the plant.
Consider a combined open loop and closed loop control system using a proportional control strategy with gain $K$ for the feedback controller as shown in the following figure. The open loop control system consists of the transfer functions $G_r(s)$, $G_{d1}^*(s)$, and $G_{d2}^*(s)$ and it is desired to have a zero error between the desired and actual velocities.

Therefore, determine the following transfer functions,

i. $G_r(s)$

ii. $G_{d1}^*(s)$

iii. $G_{d2}^*(s)$

Hint: In the solution of the each part, you do not have to determine the all plant and disturbance transfer functions, namely, $G_u(s)$, $G_{d1}(s)$, and $G_{d2}(s)$.