1) a) As the initial step of the control system design and analysis, the mathematical model of the system is obtained by writing the elemental and structural equations in Laplace domain in the following table.

<table>
<thead>
<tr>
<th>Elemental Equations:</th>
<th>Structural Equations:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hydraulic</strong></td>
<td><strong>Hydraulic</strong></td>
</tr>
<tr>
<td>(Q_L = K_v X_v - K_p P_L)</td>
<td>(Q_c = Q_L - Q_p)</td>
</tr>
<tr>
<td>(Q_c = C_s P_L)</td>
<td></td>
</tr>
<tr>
<td>(F_m = ms^2 X)</td>
<td>(F_k = k(X - Y))</td>
</tr>
<tr>
<td>(F_b = bs(X - Y))</td>
<td></td>
</tr>
<tr>
<td><strong>Hydraulic Actuator</strong></td>
<td></td>
</tr>
<tr>
<td>(F_p = A P_L)</td>
<td></td>
</tr>
<tr>
<td>(Q_p = A(sX - sY))</td>
<td></td>
</tr>
</tbody>
</table>

b) Complete the block diagram representation of the open loop system is given as follows.

[Diagram of the block diagram representation of the open loop system]
c) Initially, the plant transfer function \( G_u(s) = G_{VX_Y}(s) \) is determined by letting \( Y(s) = 0 \).

\[
G_u(s) = G_{VX_Y}(s) = \frac{V(s)}{X_Y(s)} = \frac{AK_Ys}{(Cs + K_p)(ms^2 + bs + k) + A^2s}
\]

Then, the order of the summation blocks of the hydraulic part is reversed and the feedback paths in the mechanical part simplified to yield to the following block diagram,

The feedback of the load pressure is then simplified and the following block diagram is obtained,

Therefore, the plant transfer function is determined as

\[
G_u(s) = \frac{AK_Ys}{Cms^3 + (mK_p + Cb)s^2 + (Ck + bK_p + A^2)s + kK_p}
\]
Similarly, the disturbance transfer function $G_d(s)$ is determined by letting $X_v(s) = 0$. 

This block diagram can be re-drawn to ease the reduction process as follows,

Next the internal feedback loop is eliminated,

Hence, the following unity feedback system is obtained.
Finally, the disturbance transfer function is gathered as,

\[
G_d(s) = \frac{V(s)}{s^2Y(s)} = \frac{(Cs + K_p)(bs + k) + A^2s}{s\{ms^2(Cs + K_p) + (Cs + K_p)(bs + k) + A^2s\}}
\]

\[
G_d(s) = \frac{Cbs^2 + (Ck + bK_p + A^2)s + kK_p}{s\{mCs^3 + (mK_p + Cb)s^2 + (Ck + bK_p + A^2)s + kK_p\}}
\]

d) The open loop control system is designed as follows,

- \( G_r(s) = \frac{1}{G_u(s)} \)

\[
G_r(s) = \frac{Cms^3 + (mK_p + Cb)s^2 + (Ck + bK_p + A^2)s + K_p}{AK_p s}
\]

- \( G_d'(s) = -\frac{G_d(s)}{G_u(s)} = \)

\[
= -\frac{Cbs^2 + (Ck + bK_p + A^2)s + kK_p}{s\{mCs^3 + (mK_p + Cb)s^2 + (Ck + bK_p + A^2)s + kK_p\}} \frac{(Cms^3 + (mK_p + Cb)s^2 + (Ck + bK_p + A^2)s + kK_p)}{AK_p s}
\]

\[
G_d'(s) = -\frac{Cbs^2 + (Ck + bK_p + A^2)s + kK_p}{AK_p s^2}
\]

The complete block diagram of the combined open loop and closed loop control system by assuming a proportional controller \( K_p \) is drawn as shown in the following figure.

e) Knowing that the velocity reference input to this control system is zero, \( V_r(s) = 0 \), the reference velocity feedforward part of the open loop controller will not contribute to the overall control input.
The relative velocity of the wind measured on the car could be written as

\[ v_{w/c} = v_w - v_c \]

The elemental equations are

- Force due to inclination  \[ f_r = K_R \beta \]
- Damping force of the air  \[ f_w = b_w (v_w - v_c) \]
- Traction force  \[ f_t = K_T \theta_T \]

The structural equations are

- \( f_m = f_t - f_r - f_w \) \((f_m = \text{Net force applied on the car mass in positive direction})\)

Where from Newton’s second law

\[ f_m = m \dot{v}_c \]

- \( f_t - f_r - f_w = m \dot{v}_c \)

By substituting elemental equations into structural equation

\[ K_T \theta_T - K_R \beta + b_w \frac{v_w}{c} = m \dot{v}_c \]

By taking the Laplace transform with zero initial conditions,

\[ m s V_c(s) = b_w V_{w/c}(s) + K_T \Theta(s) - K_R B(s) \]

The block diagram of the system can be obtained as
The plant equation is

\[ V_c(s) = G_u(s) \Theta_T(s) + G_{d1}(s)B(s) + G_{d2}(s)V_{w/c}(s) \]

Then, substituting the controller equation into the plant equation

\[ \Theta_T(s) = G_r(s)R(s) + G_{d1}^*(s)B^*(s) + G_{d2}(s)V_{w/c}(s) + G_c(s)(R(s) - V_c(s)) \]

Combined controller equations is

\[ G_u(s) = \frac{K_T}{ms} \]

\[ G_{d1}(s) = -\frac{K_R}{ms} \]

\[ G_{d2}(s) = \frac{b_w}{ms} \]
\[ V_c(s) = G_u(s)\left[G_r(s)R(s) + G_{d1}^*(s)B^*(s) + G_{d2}^*(s)V_{w/c}^*(s) + G_c(s)\left(R(s) - V_c(s)\right)\right] \]
\[ + G_{d1}(s)B(s) + G_{d2}(s)V_{w/c} \]

\[ V_c(s) = \frac{G_u(s)(G_r(s) + G_c(s))}{1 + G_u(s)G_c(s)} R(s) + \frac{G_u(s)G_{d1}^*(s)}{1 + G_u(s)G_c(s)} B^*(s) + \frac{G_u(s)G_{d2}^*(s)}{1 + G_u(s)G_c(s)} V_{w/c}^*(s) \]
\[ + \frac{G_{d1}(s)}{1 + G_u(s)G_c(s)} B(s) + \frac{G_{d2}(s)}{1 + G_u(s)G_c(s)} V_{w/c}(s) \]

The estimation errors are
\[ B'(s) = B(s) - B^*(s) \text{ and } V_{w/c}^* = V_{w/c} - V_{w/c}^*(s) \]

Thus, \( B(s) \) can be written as
\[ B(s) = B'(s) + B^*(s) \]

And \( V_{w/c}(s) \) can be written as
\[ V_{w/c}(s) = V_{w/c}'(s) + V_{w/c}^*(s) \]

Finally, the following equation is determined,
\[ V_c(s) = \frac{G_u(s)(G_r(s) + G_c(s))}{1 + G_u(s)G_c(s)} R(s) + \frac{G_u(s)G_{d1}^*(s)}{1 + G_u(s)G_c(s)} B^*(s) + \frac{G_u(s)G_{d2}^*(s)}{1 + G_u(s)G_c(s)} V_{w/c}^*(s) \]
\[ + \frac{G_{d1}(s)}{1 + G_u(s)G_c(s)} [B'(s) + B^*(s)] + \frac{G_{d2}(s)}{1 + G_u(s)G_c(s)} [V_{w/c}'(s) + V_{w/c}^*(s)] \]

After simplification the following equation is determined,
\[ V_c(s) = \frac{G_u(s)(G_r(s) + G_c(s))}{1 + G_u(s)G_c(s)} R(s) + \frac{G_u(s)G_{d1}^*(s) + G_{d1}(s)}{1 + G_u(s)G_c(s)} B^*(s) \]
\[ + \frac{G_u(s)G_{d2}^*(s) + G_{d2}(s)}{1 + G_u(s)G_c(s)} V_{w/c}^*(s) + \frac{G_{d1}(s)}{1 + G_u(s)G_c(s)} B'(s) \]
\[ + \frac{G_{d2}(s)}{1 + G_u(s)G_c(s)} V_{w/c}'(s) \]

Where \( V_{w/c}'(s) \) and \( B'(s) \) are equal to zero (ideal sensors are used for the disturbance estimation)

\[ V_c(s) = \frac{G_u(s)(G_r(s) + G_c(s))}{1 + G_u(s)G_c(s)} R(s) + \frac{G_u(s)G_{d1}^*(s) + G_{d1}(s)}{1 + G_u(s)G_c(s)} B^*(s) \]
\[ + \frac{G_u(s)G_{d2}^*(s) + G_{d2}(s)}{1 + G_u(s)G_c(s)} V_{w/c}^*(s) \]

In order to satisfy the zero erro, \( \theta(s) = R(s) \) should be satisfied.
The transfer function between the input and the output should be equal to one

\[ G_{\theta R}(s) = 1 \rightarrow \frac{G_u(s)(G_r(s) + G_c(s))}{1 + G_u(s)G_c(s)} = 1 \rightarrow G_r(s) = \frac{1}{G_u(s)} \]

The transfer function between the input and the wind disturbance should be zero

\[ G_{\theta W}(s) = 0 \rightarrow \frac{G_u(s)G_d^*(s) + G_d^1(s)}{1 + G_u(s)G_c(s)} = 0 \rightarrow G_d^*(s) = -\frac{G_d^1(s)}{G_u(s)} \]

The transfer function between the input and the disturbance due to inlinaction should be zero

\[ G_{\theta S}(s) = 0 \rightarrow \frac{G_u(s)G_d^1(s) + G_d(s)}{1 + G_u(s)G_c(s)} = 0 \rightarrow G_d^1(s) = -\frac{G_d(s)}{G_u(s)} \]

Therefore the transfer functions of the combined open loop controller is determined as

\[ G_r(s) = \frac{1}{K_T/m} = \frac{m}{K_T} \]

\[ G_d^1(s) = \frac{K_R/m}{K_T} = \frac{m}{K_T} \]

\[ G_d^2(s) = -\frac{b_w}{mK_T} = -\frac{b_w}{K_T} \]