SOLUTIONS OF MIDTERM EXAMINATION I

April 19, 2010
Time Allowed: 100 minutes
Open Notes and Books
All questions are equally weighted

Student No. : ....................
Name : ............................
SURNAME : ........................
Signature : ........................

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PROBLEM 1

A typical hydraulic damping device is shown in the following figure. It is desired to obtain a mathematical modeling of this system, which takes into account the following effects:

- $m_1$ : combined mass of the piston and its connecting rod
- $m_2$ : combined mass of the cylinder body and its connecting rod
- $R$ : fluid resistance of the orifice connecting two chambers due to viscosity of hydraulic fluid
- $C_1$ : fluid capacitance of the chamber 1 due to the compressibility of hydraulic fluid
- $C_2$ : fluid capacitance of the chamber 2 due to the compressibility of hydraulic fluid
- $f_1(t)$ and $f_2(t)$ : external forces applied to two ends of this device as the only inputs.

Note that the net cross sectional areas of the piston and cylinder in side 1 and side 2 (which are subject to pressures of chambers 1 and 2) are different due to non-negligible area of the piston rod.

a) Draw the free body diagram of the piston-rod assembly using the variables shown in the block diagram, only.

b) In the modeling process of this device in Laplace domain, some of the equations are already supplied. Give the missing equations by using the variables associated with related blocks or summing points in the block diagram, only.

<table>
<thead>
<tr>
<th>Elemental Equations</th>
<th>Structural Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{m1} = m_1 s V_1$</td>
<td>$F_{p1} = A_1 p_1$</td>
</tr>
<tr>
<td>$F_{m2} = m_2 s V_2$</td>
<td>$Q_{p1} = A_1 V_{21}$</td>
</tr>
<tr>
<td>$Q_{c1} = C_1 s P_1$</td>
<td>$F_{p2} = A_2 P_2$</td>
</tr>
<tr>
<td>$Q_{c2} = C_2 s P_2$</td>
<td>$Q_{p2} = A_2 V_{21}$</td>
</tr>
<tr>
<td>$P_R = R Q_R$</td>
<td>$F_{m2} = -F_2 - F_{p1} + F_{p2}$</td>
</tr>
<tr>
<td></td>
<td>$Q_{c1} = Q_{p1} + Q_R$</td>
</tr>
<tr>
<td></td>
<td>$Q_{c2} = -Q_{p2} - Q_R$</td>
</tr>
<tr>
<td></td>
<td>$P_R = P_2 - P_1$</td>
</tr>
<tr>
<td></td>
<td>$V_{21} = V_2 - V_1$</td>
</tr>
</tbody>
</table>

Part (a) mechanical sub–system

Part (c) Complete the following detailed block diagram representation of the system by writing appropriate transfer functions inside the operational blocks.

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ME304(Sections01,02,04)/11S/MT1-Solutions/Balkan,Platin,Yazıcıoğlu
PROBLEM 2

Consider the following feedback control system with P-controller for an electric vehicle powered by a DC motor. The aim is to keep the vehicle speed \( v \), at a desired level \( v_r \) on an inclined road with percent grade \( \beta \) (a simple cruise controller). The speed sensor has a transfer function \( 1/(Ts+1) \) as shown in the figure.

- **U(s)** : Voltage applied to DC motor
- **\( \tau(s) \)** : Torque generated by the DC motor
- **m** : Total mass of the vehicle
- **r** : Wheel radius
- **T** : Time constant of the speed sensor
- **\( K_u, K_\omega \)** : DC motor parameters
- **K** : Controller gain
- **\( K_r \)** : Disturbance gain (Car weight, mg)

a) Design a feedforward controller by obtaining relevant transfer functions that will result in zero error between the desired and actual velocities. Note that this feedforward controller assumes \( v_r \) and \( \beta^* \) (measurement of \( \beta \)) as inputs to produce the open loop contribution of \( u \). Note also that the feedback controller with speed sensor should not be considered in the design of such a feedforward controller.

b) Redraw the complete block diagram representation of the combined feedback and feedforward control system if \( B \) is measured with an ideal (i.e., with unity gain) inclinometer.

**Solution:**

a) If the plant open-loop transfer functions, \( G_u \) and \( G_d \) are determined then \( G_u(s) = \frac{1}{G_u(s)} \) and

\[
G_u(s) = -\frac{G_d(s)}{G_u(s)}
\]
\[
\left(\frac{K_u U - \frac{K_m}{r} V}{r} - K_r B\right) \frac{1}{ms} = V \rightarrow rK_u U - r^2 K_r B = \left(mr^2 s + K_m\right)V
\]

\[
V = \frac{rK_u}{mr^2 s + K_m} U + \frac{-r^2 K_r}{mr^2 s + K_m} B. \quad \text{So} \quad G_u(s) = \frac{rK_u}{mr^2 s + K_m} \quad \text{and} \quad G_d(s) = \frac{-r^2 K_r}{mr^2 s + K_m}
\]
or alternatively by using block diagram algebra

\[
\begin{align*}
G_u(s) &= \frac{V(s)}{U(s)} = \frac{rK_u}{mr^2 s + K_m} \\
G_d(s) &= \frac{V(s)}{B(s)} = \frac{-r^2 K_r}{mr^2 s + K_m}
\end{align*}
\]

Thus, \( G_r(s) = \frac{1}{G_u(s)} = \frac{mr^2 s + K_m}{rK_u} \) and \( G_d(s) = -\frac{G_d(s)}{G_u(s)} = \frac{r^2 K_r}{rK_u} = \frac{rK_r}{K_u} \)

b) For a combined feedback and feedforward control system

\[
G_r(s) = \frac{1}{G_u(s)} + KG_c(s)[H(s) - 1] \quad \text{and} \quad G_d(s) = -\frac{G_d(s)}{G_u(s)} \quad \text{where} \quad G_c(s) = 1 \quad \text{and} \quad H(s) = \frac{1}{Ts+1}.
\]

\[
G_r(s) = \frac{1}{G_u(s)} + KG_c(s)[H(s) - 1] = \frac{mr^2 s + K_m}{rK_u} + K\left(\frac{1}{Ts+1} - 1\right) = \frac{mr^2 s + K_m}{rK_u} + \frac{KT_s}{Ts+1}
\]

Block diagram representation of the combined control system
PROBLEM 3

The equivalent block diagram of a DC-motor in torque control mode and its load is given below. The input to the system is the reference voltage input $V_{tr}$ corresponding to the desired torque and the output of the system is the motor shaft angular velocity, $\Omega$. The torque of the motor is estimated through the armature current, $I_a$ which is measured as voltage using a current sensing resistor, $R_i$, connected in series with the armature resistance, $R_a$.

The transfer functions in the block diagram are as follows:

$$
G_1 = K_c \\
G_2 = \frac{1}{L_a s + R_a} \text{ for small } R_a \\
G_3 = K_t \\
G_4 = \frac{1}{J s + b} \\
H_1 = K_b \\
H_2 = R_i
$$

a) Use graphical block diagram reduction method to obtain the closed loop transfer function $\frac{\Omega(s)}{V_{tr}(s)} = M(s)$ in terms of system and controller parameters.

b) The performance of the control system will be dependent on the uncertainty of the current sensing resistor. Determine the uncertainty of the closed loop transfer function $\delta M$ based on the uncertainty of the current sensing resistor $\delta R_i$ under steady state conditions. Note that, at steady state, $M_{ss}$ can be written as

$$
M_{ss} = \frac{K_c K_t}{R_a b + K_t K_b + K_c R_i b}
$$

Solution:

a)
\[
\frac{\Omega(s)}{V_{tr}(s)} = M(s) = \frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4H_1 + G_2G_3G_4H_1}
\]
Substitute for \( G_1, G_2, G_3, G_4, H_1, \) and \( H_2. \)

\[
\frac{\Omega(s)}{V_{tr}(s)} = M(s) = \frac{K_c \frac{1}{L_a s + R_a} K_i \frac{1}{Js + b}}{1 + K_c \frac{1}{L_a s + R_a} R_i + \frac{1}{L_a s + R_a} K_t \frac{1}{Js + b} K_b}
\]

Simplify \( M(s) \) and find the result.

\[
\frac{\Omega(s)}{V_{tr}(s)} = M(s) = \frac{K_c K_t}{L_a J s^2 + (L_a b + R_a J + K_c R_i J)s + (R_a b + K_t K_b + K_c R_i b)}
\]

b) \[
\frac{\partial M_{ss}}{M_{ss}} = S_{R_i}^{M_{ss}} \frac{\partial R_i}{R_i} = \frac{R_i}{M_{ss}} \frac{\partial M_{ss}}{\partial R_i} \frac{\partial R_i}{R_i} = \frac{\partial M_{ss}}{\partial R_i} \frac{\partial R_i}{R_i}
\]

\[
\frac{\partial M_{ss}}{M_{ss}} = S_{R_i}^{M_{ss}} \frac{\partial R_i}{R_i} = \frac{R_i}{M_{ss}} \frac{\partial M_{ss}}{\partial R_i} \frac{\partial R_i}{R_i} = \frac{\partial M_{ss}}{\partial R_i} \frac{\partial R_i}{R_i}
\]

\[
= -\frac{K_c b (K_c K_i)}{(R_a b + K_i K_b + K_c R_i b)^2} \partial R_i = -\frac{K_c^2 K_i b}{(R_a b + K_i K_b + K_c R_i b)^2} \partial R_i
\]