ME 304 CONTROL SYSTEMS  
Spring 2005  
Sections 02, 03, 04  

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SOLUTIONS TO MIDTERM EXAMINATION I  

April 09, 2005  
Time Allowed: 100 minutes  
Open Notes and Books  
All questions are equally weighted  

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**PROBLEM 1:**

Consider the following hydraulic valve and actuator model. For simplicity, the valve orifices are modeled as linear fluid resistances $R_1$ and $R_2$. The hydraulic oil is compressible and has bulk modulus $\beta$. $V_0$ is the average volume of the pressurized actuator chamber.

\[ m, b, k, A : \text{actuator parameters} \]
\[ R_1, R_2 : \text{linear valve orifice resistances} \]
\[ p_s(t) : \text{controlled pump pressure} \]
\[ p_e(t) : \text{return pressure} \]
\[ p(t) : \text{pressure in actuator chamber} \]
\[ x(t) : \text{actuator piston position} \]

A detailed but incomplete block diagram representation of the system is given below.

a) In the space provided, write down the equations such that each equation will correspond to one operational block or summing point in the block diagram.

\[ f_m = m\ddot{x} \quad (1) \]
\[ f_b = b\dot{x} \quad (2) \]
\[ f_k = kx \quad (3) \]
\[ f_p = Ap \quad (4) \]
\[ f_m = f_p - f_b - f_k \quad (5) \]
\[ \Delta p = p_s - p \quad (6) \]
\[ v = \dot{x} \quad (7) \]
\[ q_1 = \frac{1}{R_1}(p_s - p) \quad (8) \]
\[ q_2 = \frac{1}{R_2}p \quad (9) \]
\[ q_p = A\dot{x} \quad (10) \]
\[ q_c = \frac{V_0}{\beta} \dot{\hat{p}} \quad (11) \]
\[ q_{c1} = q_{1} - q_{2} - q_{p} \quad (12) \]

b) Complete the given detailed block diagram representation of this system by writing appropriate transfer functions inside the operational blocks. Also, name the branches of the block diagram by using the variables you have used in part (a).
PROBLEM 2:

For the following hydro-mechanical system,

the mathematical representation is given by the following set of equations:

\[
\begin{align*}
\dot{Q}_A &= A\dot{v} \quad (2) \\
Q_a &= a\dot{v} \quad (5) \\
Q_c + Q_{in} &= C\dot{p}_1 \quad (7) \\
p_1 &= R_1Q_1 \quad (8) \\
p_1 &= R_2Q_2 \quad (9)
\end{align*}
\]

Determine the transfer functions between
i) the pressure \(p_1\) at the bottom of the tank and \(Q_{in}(t)\), and
ii) the total discharge to the environment, that is \(Q_{tot}=Q_1+Q_2\) and \(p_s(t)\).

as ratios of two polynomials in \(s\).

Solution:

Taking the Laplace transform the set of equations given above yields

\[
\begin{align*}
\dot{F}_A &= AP_s \quad (1) \\
F_A - F_a &= m\dot{v} \quad (3) \\
F_a &= aP_1 \quad (4) \\
Q_a &= Q_c + Q_1 + Q_2 \quad (6) \\
P_1 &= R_1Q_1 \quad (8) \\
P_1 &= R_2Q_2 \quad (9)
\end{align*}
\]

a) Defining \(P_1(s)\) as the output requires the elimination of rest of the system variables.

Using (5), (7), (8), and (9) in (6) gives

\[
a\dot{V} = (CsP_1 - Q_{in}) + \frac{P_1}{R_1} + \frac{P_1}{R_2} \quad (10)
\]

Multiplying (10) by \(R_1R_2\) and using (3) in (10) gives

\[
aR_1R_2 \frac{1}{ms}(F_A - F_a) = R_1R_2CsP_1 + (R_1 + R_2)P_1 - R_1R_2Q_{in} \quad (11)
\]

Multiplying (11) by \(ms\) and using (1) and in (10) gives

\[
aR_1R_2(AP_s - aP_1) = R_1R_2Cs^2P_1 + (R_1 + R_2)msP_1 - R_1R_2msQ_{in} \quad (12)
\]

Rearranging (12) gives

\[
[R_1R_2Cs^2 + (R_1 + R_2)ms + a^2R_1R_2]P_1 = R_1R_2aAP_s + R_1R_2msQ_{in} \quad (13)
\]

Hence the transfer function between \(P_1(s)\) and \(Q_{in}(s)\) becomes

\[
G_{P_1Q_{in}}(s) = \frac{R_1R_2ms}{R_1R_2Cs^2 + (R_1 + R_2)ms + a^2R_1R_2}
\]
b) Using (8) and (9) in \( Q_{tot} = Q_1 + Q_2 \), one gets

\[
Q_{tot} = \frac{P_1}{R_1} + \frac{P_1}{R_2} = \frac{R_1 + R_2}{R_1R_2} P_1 \tag{14}
\]

Using (14) in (13) gives

\[
[R_1R_2Cms^2 + (R_1 + R_2)ms + a^2R_1R_2]Q_{tot} = (R_1 + R_2)aAP_s + (R_1 + R_2)msQ_{in} \tag{15}
\]

Hence the transfer function between \( Q_{tot}(s) \) and \( P_s(s) \) becomes

\[
G_{Q_{tot}P_s}(s) = \frac{(R_1 + R_2)aA}{R_1R_2Cms^2 + (R_1 + R_2)ms + a^2R_1R_2}
\]
PROBLEM 3:

In the system shown above, a polishing tool of mass \( m \) is pressed on a moving plate to be polished by means of a displacement actuator that transmits its action through a spring of stiffness \( k \). The displacement of the actuator is denoted by \( y \).

It is required that the contact force \( N \) between the tool and the plate be kept close to a desired value \( N_r \) as much as possible, even though the plate has a certain motion denoted by \( x \). The motion of the plate is measured with a noisy displacement sensor whose output is \( z = x + w_x \), where \( w_x \) is the measurement noise.

Note that the contact force \( N \) can be expressed as

\[
N(s) = kY(s) - (ms^2 + k)X(s)
\]

a) Let the control input \( Y(s) \) be generated using an open-loop controller as

\[
Y(s) = G_y(s)Z(s) + G_r(s)N_r(s)
\]

Determine \( G_y(s) \) and \( G_r(s) \) properly so that the actual error becomes

\[
E(s) = N_r(s) - N(s) = -(ms^2 + k)W_x(s)
\]

b) Let the controller be improved by adding a PI feedback control action so that

\[
Y(s) = G_y(s)Z(s) + G_r(s)N_r(s) + G_f(s)E'(s)
\]

with \( G_f(s) = K_p(1 + \frac{1}{Ts}) \)

Here, \( E'(s) = N_r(s) - N'(s) \) is the actuating error and \( N'(s) \) is the output of a noisy force sensor with a measurement noise \( w_n \) such that

\[
N'(s) = N(s) + W_n(s)
\]

Again, determine \( G_y(s) \) and \( G_r(s) \) properly and show that the actual error can now be expressed as

\[
E(s) = G_n(s)W_n(s) - G_x(s)W_x(s).
\]

c) Considering the transfer functions \( G_n(s) \) and \( G_x(s) \), determine which one of \( W_n(s) \) and \( W_x(s) \) has a suppressible effect on \( N(s) \) and explain clearly how its effect can be suppressed.

Solution:

It is given that

\[
N(s) = kY(s) - (ms^2 + k)X(s).
\]

\[
Y(s) = G_y(s)Z(s) + G_r(s)N_r(s).
\]
a) With the necessary substitutions,

\[ N(s) = N_r(s) - E(s) = k[G_z(s)Z(s) + G_r(s)N_r(s)] - (ms^2 + k)[Z(s) - W_x(s)], \]

\[ E(s) = [(ms^2 + k) - kG_z(s)]Z(s) + [1 - kG_r(s)]N_r(s) - (ms^2 + k)W_x(s). \]

The effects of \( N_r(s) \) and \( Z(s) \) can be eliminated by

\[ G_r(s) = 1/k \quad \text{and} \quad G_z(s) = (ms^2 + k)/k = 1 + (m/k)s^2. \]

Hence, the actual error becomes

\[ E(s) = N_r(s) - N(s) = -(ms^2 + k)W_x(s). \]

b) Noting that

\[ E' = N_r - N' = N_r - (N + W_n) = (N_r - N) - W_n = E - W_n, \]

and using the same open-loop control functions, \( E(s) \) becomes

\[ E(s) = -(ms^2 + k)W_x(s) - kG_f(s)[E(s) - W_n(s)], \]

\[ [1 + kG_f(s)]E(s) = kG_f(s)W_n(s) - (ms^2 + k)W_x(s), \]

\[ [Ts + kK_p(Ts + 1)]E(s) = kK_p(Ts + 1)W_n(s) - Ts(ms^2 + k)W_x(s), \]

\[ E(s) = \frac{kK_p(Ts + 1)}{(1 + kK_p)Ts + kK_p}W_n(s) - \frac{Ts(ms^2 + k)}{(1 + kK_p)Ts + kK_p}W_x(s). \]

c) As seen above, if \( K_p \) can be made very large, \( E(s) \) becomes

\[ E(s) \approx W_n(s). \]

In other words, as expected, a very large feedback control gain can suppress the effect of \( W_x(s) \) (which is the noise in the disturbance) but not the effect of \( W_n(s) \) (which is the noise in the feedback signal).