ME 304 CONTROL SYSTEMS
Spring 2004
Sections 01, 03, 04

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SOLUTIONS TO MIDTERM EXAMINATION I

April 12, 2004
Time Allowed: 75 minutes
Open Notes and Books
All questions are equally weighted

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**PROBLEM 1:**

For the system represented by the block diagram given below find the following transfer functions:

i) \( G_{CN}(s) \)  
ii) \( G_{CR}(s) \)  
iii) \( G_{AN}(s) \)  
iv) \( G_{AR}(s) \)

(Your answers must be given as ratios of two expanded expressions in terms of transfer functions \( G_1, G_2, G_3, G_4, G_5, \) and \( H_1 \), without any parentheses.)

**SOLUTION:**

Elimination of the only loop gives

\[
\begin{align*}
\text{i) } G_{CN}(s) &= G_5 \\
\text{ii) } G_{CR}(s) &= G_4 + G_1 \frac{G_2}{1 + G_2 H_1} G_3 = \frac{G_4 + G_4 G_2 H_1 + G_1 G_2 G_3}{1 + G_2 H_1} \\
\text{iii) } G_{AN}(s) &= 0 \\
\text{iv) } G_{AR}(s) &= G_1 \frac{G_2}{1 + G_2 H_1} G_3 = \frac{G_1 G_2 G_3}{1 + G_2 H_1}
\end{align*}
\]
PROBLEM 2:

The system shown above is a physical model of the drive system of a lathe and consists of an actuator, a long elastic shaft, a rigid lead screw, and a carriage attached to the nut of the screw. The torsional stiffness of the shaft is $k_t$. The mass of the carriage is $m$. All the frictional effects are modeled as if they are accumulated in the bearing at the right end of the screw as a rotational viscous friction with coefficient $b_t$. The pitch of the screw is $\lambda$, which is defined so that $\dot{x}(t) = \lambda \dot{\phi}(t)$, where $x(t)$ is the linear displacement of the carriage and $\phi(t)$ is the angular displacement of the of the screw. The inputs of the system are specified as the angular displacement $\theta(t)$ supplied to the left end of the long shaft by the actuator and the force $F(t)$ applied externally upon the carriage.

a) Draw the free body diagrams for the elastic shaft, for the lead screw, and the nut+carriage combination. Show the torque and force interactions clearly and consistently. Denote these interactions with the symbols $T_k$, $T_b$, $T_\lambda$, and $F_\lambda$.

b) Write all the six elemental and structural equations required for a complete mathematical description of this system.

c) Derive the $s$-domain equation that relates the output $X(s) = \mathcal{L}\{x(t)\}$ to the inputs $\Theta(s) = \mathcal{L}\{\theta(s)\}$ and $F(s) = \mathcal{L}\{F(t)\}$.

SOLUTION:

See next page
SOLUTION TO PROBLEM 2

a) Free Body Diagrams:

b) Elemental and Structural Equations:

- Elastic Shaft: \( T_k = k_t (\theta - \phi) \). (Eqn. 1)
- Torsional Damper: \( T_b = b_t \phi \). (Eqn. 2)
- Carriage: \( ma = m\ddot{x} = F_{\lambda} - F \). (Eqn. 3)
- Screw-Bolt Pair: \( \dot{x} = \lambda \dot{\phi} \rightarrow F_{\lambda} = (1/\lambda)T_{\lambda} \). (Eqns. 4 and 5)
- Structural Eqn: \( T_k = T_{\lambda} + T_b \). (Eqn. 6)

c) Input-Output Equation in the s-Domain:

\[
\begin{align*}
T_k(s) &= k_t[\Theta(s) - \Phi(s)], \\
T_b(s) &= b_t s \Phi(s), \\
X(s) &= \lambda \Phi(s), \\
F_{\lambda}(s) &= T_{\lambda}(s)/\lambda, \\
m^2 s^2 X(s) &= F_{\lambda}(s) - F(s), \\
T_k(s) &= T_{\lambda}(s) + T_b(s). \\
\end{align*}
\]
SOLUTION TO PROBLEM 3:

Consider the following quarter car model with active suspension control. The differential equation describing the dynamic behavior of this system can be represented as

\[ m\ddot{x} + b\dot{x} + kx = b\dot{y}(t) + ky(t) + f(t) \]

where
- \( m, b, k \): car and suspension system parameters
- \( y(t) \): road disturbance
- \( x(t) \): driver’s vertical position
- \( v(t) \): driver’s vertical velocity, i.e. \( v(t) = \dot{x}(t) \)
- \( f(t) \): actuator force

a) The block diagram representation of the system is shown below at left. Complete the alternative equivalent block diagram representation at right by writing transfer functions inside the operational blocks.

b) Consider the speed control system for a better driving comfort. If the driver’s desired vertical speed is \( v_r(t) \equiv 0 \), design an open-loop speed controller and complete the following block diagram assuming that the disturbance is measured with an ideal accelerometer (acceleration measuring device).

c) Now, consider a combined open-loop and closed-loop control of the system using a proportional control strategy with gain \( K \) is selected for the feedback controller while the vertical accelerations of the driver and the disturbance are measured with ideal accelerometers. The feedforward reference velocity part of the open-loop controller will not be used since the driver’s desired vertical speed is zero. Complete the following block diagram.