PROBLEM 1

Analytically determine all possible kinematic chains which have the following properties:

i) Three degrees of freedom and planar.

ii) Composed of 9 links with 1 regular gear pair, 1 prismatic joint and the remaining being revolute joints.

iii) Does not contain any links with 5 or more kinematic pairs.

Find the number of joins (j). Derive the equations for the number of binary and ternary links ($l_2, l_3$) in terms of the number of quaternary links ($l_4$). For each possible kinematic chain, determine $l_2, l_3$ and $l_4$. Sketch two of the kinematic chains with distinct characteristics.

SOLUTION

Design requirements are:

3 degrees of freedom: $F = 2$; Planar: $\lambda = 3$; 9 links: $l = 9$

Let $j$ be the total number of joints. Then, the number of revolute joints in the mechanism becomes $(j - 2)$. So, considering $(j - 2)$ number of revolute, 1 prismatic joints and 1 regular gear pair,

$$\sum f_j = (j - 2).1 + (1).1 + (1).2 = (j + 1)$$

Substituting the known parameters into the degree of freedom formula yields;

$$F = \lambda(l - j - 1) + \sum f_j$$

$$3 = 3(9 - j - 1) + (j + 1) \rightarrow [j = 11]$$

Recall the equations relating the number of binary, ternary and quaternary links:

$$l_2 + l_3 + l_4 = l = 9$$

$$2l_2 + 3l_3 + 4l_4 = 2j = 22$$

Solution of the previous equations for the number of binary and ternary links ($l_2, l_3$) in terms of the number of quaternary links ($l_4$),

$$l_3 = 4 - 2l_4 \text{ and } l_2 = 5 + l_4$$

<table>
<thead>
<tr>
<th></th>
<th>$l_4$</th>
<th>$l_3$</th>
<th>$l_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Case 2</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Case 3</td>
<td>2</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
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Note that the number of links cannot be negative. Therefore, there are 3 distinct cases.
Note that; gears are ternary links in case 1&2 and binary link in case 3.

**PROBLEM 2**

For the mechanisms given:

i) Count the number of links \((l)\), joints \((j)\) and determine the connectivity sum \((\sum f_i)\). Determine the degree of freedom (DOF) of the mechanism (if the question has not been asked in HW1).

ii) Draw simplified sketch of the mechanism.

iii) Show all necessary fixed parameters (or dimensions) and joint variables clearly on the sketch.

iv) Find the number of independent loops.

v) Select the joint(s) to be virtually disconnected and write down the loop closure equations both as a vector equation (e.g. \(\vec{AB} + \vec{BC} = \vec{AC}\)) and as a complex equation (e.g. \(|AB|e^{i\theta_{12}} + |BC|e^{i\theta_{13}} = |AC|e^{i\theta_{13}}\)).

vi) Make a list of fixed parameters and the joint variables that you have used.

Use \(\theta_{ij} \& s_{ij}\) for the joint variables; \(|AB|\), \(|BC|\), etc. for the constant lengths and \(\alpha_i, \beta_i\) etc. for the constant angles.

a)
• \( \lambda=3, l=6, j=7 \) (6R,1P)
  \[ F = 3(6 - 7 - 1) + 7 = 1 \]
• \( L = j - l + 1 \rightarrow L = 7 - 6 + 1 \rightarrow L = 2 \)
• Fixed Parameters: \( |AH|, |HF|, |FG|, |GD|, |BD|, |DC|, |EC|, |FE|, \gamma_4 \)
• Joint Variables: \( s_{23}, \theta_{12}, \theta_{14}, \theta_{15}, \theta_{16} \)
• Virtually disconnect point B (Origin at A):
  \[ \overline{AB} = \overrightarrow{AH} + \overrightarrow{HF} + \overrightarrow{FG} + \overrightarrow{GD} + \overrightarrow{DB} \]
  \[ s_{23}e^{i\theta_{12}} = -i|AH| + |HF| + i|FG| + |GD| + |DB|e^{i(\theta_{14} + \gamma_4)} \]
• Virtually disconnect point C (Origin at D):
  \[ \overline{DC} = \overrightarrow{DG} + \overrightarrow{GF} + \overrightarrow{FE} + \overrightarrow{EC} \]
  \[ |DC|e^{i\theta_{14}} = -|DG| - i|GF| + |FE|e^{i\theta_{16}} + |EC|e^{i\theta_{15}} \]
• \( \lambda = 3, l = 7, j = 9 \) (7R, 1P, 1G)
  \( F = 3(7 - 9 - 1) + 10 = 1 \)
• \( L = j - l + 1 \rightarrow L = 9 - 7 + 1 \rightarrow L = 3 \)
• Fixed Parameters: \( |AB|, |BC|, |CD|, |DE|, |AH|, |AK|, |HG|, |GF|, |FE| \)
• Joint Variables: \( s_{17}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{16} \)
• Virtually disconnect point C (Origin at A):
  \[
  \bar{AB} + \bar{BC} = \bar{AK} + \bar{KD} + \bar{DC}
  \]
  \[
  |AB|e^{i\theta_{12}} + |BC|e^{i\theta_{13}} = |AK| + is_{17} + |CD|e^{i\theta_{14}}
  \]
• Virtually disconnect point E (Origin at K):
  \[
  \bar{KD} + \bar{DE} = \bar{KG} + \bar{GF} + \bar{FE}
  \]
  \[
  is_{17} + |DE|e^{i(\theta_{14} + \pi)} = [|AH| + |HG| - |AK|] + |GF|e^{i\theta_{15}} + |FE|e^{i\theta_{16}}
  \]
For the gear pair,
  \[
  |AH|\Delta \theta_{12} = |HG|\Delta \theta_{16}
  \]