Pr. 1) In order to examine the crack tip fields for a mode II crack, we consider a semi – infinite crack in infinite plane loaded by a pair of horizontal forces of magnitude $Q$. Shear modulus and Poisson’s ratio of the material are given as $\mu$ and $\nu$, respectively. For this skew-symmetric problem the Airy stress function is given as:

$$\Phi(x, y) = -y \text{Re}(\bar{Z})$$

where $\frac{d\bar{Z}}{dz} = Z$, $\frac{dZ}{dz} = Z'$, and all are analytic complex functions. Notice that these functions may have isolated singularities in the complex plane.

Semi – infinite crack in infinite plane loaded by a pair of tangential forces

a) Show that the given Airy stress function satisfies the biharmonic equation.

b) Derive the expressions for the planar stress components $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ in terms of the analytic complex functions given above.

c) Show that the Westergaard stress function

$$Z(z) = \frac{Q}{\pi(z + b)} \sqrt{\frac{b}{z}}$$

satisfies all boundary conditions of the problem.
d) Using the Westergaard function given in part (c), derive the expressions for stress components valid everywhere throughout the elastic body.

e) Plot $\sigma_{yy}(x,0)$.

f) Derive the asymptotic expressions for the stress components in the vicinity of the crack tip $x = a$, for small values of $r$.

\[
\begin{align*}
&\text{r} \\
&a \\
-&a &a
\end{align*}
\]

Polar coordinate system at the crack tip

g) Find the expression of the mode II stress intensity factor $K_{II}$.

Pr. 2) Consider Case 5.11 given in the Compendium of Westergaard Stress Functions and assume that loading is mode I, i.e. assume that $Q = 0$.

a) Determine $\sigma_{yy}$ distribution on the crack plane.

b) Derive the expressions of the mode I stress intensity factors.